Mathematics	Mid-Term Exam	November 2015	Total Mark: 20

[1]Find y\` :

(a) $y = 3x^2 + 2^x - 2x$ (b) $y = \ln x . \log x + 3$

(c) $y = \sin x . \cos x + (\sin x)^4$ (d) $y = \frac{5}{x^5} + \frac{3^x}{2^x}$

(e) $y = \ln x + \log x$ (f) $y = \ln x^5 + \log(\sin x + \cos x)$

Solution

(a) $y` = 6x + 2^x . ln\, 2 - 2$

(b) $y` = ln\, x . \frac{1}{ln\, 10} . \frac{1}{x} + \frac{1}{x} . log\, x + 0$

(c) $y` = sin\, x . (-\sin x) + \cos x . cos\, x + 4(sin\, x)^3 \cos x$

(d) $y = 5x^{-5} + (\frac{3}{2})^x$, $y` = -25x^{-6} + \left(\frac{3}{2}\right)^x . ln(\frac{3}{2})$

(e) $y` = \frac{1}{x} + \frac{1}{ln\, 10} . \frac{1}{x}$

(f) $y` = \frac{5}{x} + \frac{1}{ln\, 10} . \frac{\cos x - \sin x}{\sin x + \cos x}$

---*6-marks*

[2]Find the extrema of each function:

(a) $f(x) = 1 - \ln x$ (b) $f(x) = x^3 - 6x^2 + 9x + 3$

Solution

(a) $f(x) = 1 - ln\, x$, $f`(x) = 0 - \frac{1}{x} \neq 0$ No extrema

---*1-mark*

(b) $f(x) = x^3 - 6x^2 + 9x + 3$, $f`(x) = 3x^2 - 12x + 9 = 0$

Then $x^2 - 4x + 3 = (x-1)(x-3) = 0$ and the critical points are 1, 3.

From $f``(x) = 6x - 12$.

Then $f``(1) = 6 - 12 = -6$, then 1 is maximum.

$f``(3) = 18 - 12 = 6$, then 3 is minimum.

---*3-marks*

[3]Find the integrals:

(a) $\int (2^x - x^2)\, dx$ (b) $\int (\cos 3x + \sin 2x)\, dx$ (c) $\int (\frac{1}{x} - \frac{3}{x+5})\, dx$

(d) $\int (x^2 - 2x)^2\, dx$ (e) $\int \frac{1}{x} (3 + \ln x)^8\, dx$ (f) $\int 2x \ln x\, dx$

Solution

(a) $\int(2^x - x^2)\,dx = \frac{2^x}{\ln 2} - \frac{x^3}{3} + c$

(b) $\int(\cos 3x + \sin 2x)\,dx = \frac{\sin 3x}{3} - \frac{\cos 2x}{2} + c$

(c) $\int(\frac{1}{x} - \frac{3}{x+5})\,dx = \ln x - 3\ln(x+5) + c$

(d) $\int(x^2 - 2x)^2\,dx = \int(x^4 + 4x^2 - 4x^3)\,dx = \frac{x^5}{5} + 4\frac{x^3}{3} - x^4 + c$

(e) $\int \frac{1}{x}(3 + \ln x)^8\,dx = \frac{1}{9}(3 + \ln x)^9 + c$

(f) $\int 2x \ln x\,dx$

By parts: Put $u = \ln x$, $dv = 2xdx$. $Then$ $u` = \frac{1}{x}$, $v = x^2$

Then $\int 2x \ln x\,dx = u.v - \int v\,du = x^2 \ln x - \int x^2.\frac{1}{x}dx = x^2 \ln x - \int x\,dx$

$$= x^2 \ln x - \frac{1}{2}x^2 + c$$

--*6-marks*

[4]If y is the quantity of a drug decreases according to the equation : $\frac{dy}{dt} = -y^{\frac{2}{3}}$.
Find y as function of the time t where the initial quantity is 27 units.
Also, find (i)The value of y after 3 hours.
(ii)The time at which there is no drug in the blood.

Solution

From the equation : $\frac{dy}{dt} = -y^{\frac{2}{3}}$, then $y^{-\frac{2}{3}}dy = -dt$

Integrate both sides, we get $\frac{y^{\frac{1}{3}}}{\frac{1}{3}} = -t + c$ or $3\sqrt[3]{y} = -t + c$

At t = 0 , y = 27, we get $3\sqrt[3]{27} = 0 + c$. Then 9 = c and $3\sqrt[3]{y} = -t + 9$

Then $\sqrt[3]{y} = \frac{1}{3}(9 - t)$ and the explicit relation is : $y = \frac{1}{27}(9 - t)^3$

(i)After 3 hours, we get $y = \frac{1}{27}(9 - 3)^3 = \frac{6.6.6}{27} = 8$ units.

(ii) There is no drug in the blood when y = 0, then $0 = \frac{1}{27}(9 - t)^3$.

Then, the time is 9 hours.

--*4-marks*

Dr. Mohamed Eid